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Comparison of Design Optimization Formulations for Minimization of Noise Transmission in a Cylinder

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A computational design tool was developed to perform a constrained optimization of the acoustic environment within a vibrating cylinder, incorporating finite element and boundary element methods. This paper presents a comparison of the relative performance and results obtained through the use of four different objective function and constraint formulations within the optimization tool: 1) minimize the sum of the squared acoustic pressures at a number of points within the cylinder, 2) minimize the weight of the structure with a constraint on the sum of the acoustic pressures, 3) minimize the sum of the acoustic pressures with a constraint on weight, and 4) minimize weight and summed pressures simultaneously. All four formulations were subject to upper and lower bounds on the design variables, the structural shell element thicknesses. All analyses were performed at a single excitation frequency. These formulations were implemented within the design tool and compared based on their overall pressure and weight reduction and computational efficiency. Two different cylinder models were used to evaluate the optimization formulations. It was found that the third formulation was the most stable and effective. In addition, a nearly linear relationship was observed between the relative optimal decibel noise level within the cylinder and the optimal cylinder weight.

Introduction

WE present an evaluation of the behavior and computational performance of four different objective and constraint formulations, implemented within a computational design tool, for the optimization of the skin thickness distribution of unstiffened cylinder models. The desired goals of the optimization are related to the structural weight and interior noise environment of the cylinder models. The cylinders are considered to be excited by an external noise source, modeled here as a monopole. The formulations are evaluated based on their effectiveness and efficiency in optimizing the structural acoustic system. The cylinder wall thicknesses are considered to be the design variables. The cylinders considered here and the exterior noise source are not representative of aircraft structures, per se, but are suitable for evaluating the relative merits of the objective formulations, which is the key focus of this paper.

There has recently been increasing interest among airlines and aircraft manufacturers in controlling the acoustic environment within aircraft cabins to provide passengers with a more comfortable environment and to provide the flight crew with a flight deck environment that does not impair communication. In turboprop aircraft, the interior noise is dominated by noise caused by the propeller blades. No current certification requirements exist to regulate interior noise, but airlines do require noise level guarantees of the manufacturer. Airframers have confronted this problem by two broad means: 1) active and 2) passive noise control. Classical active noise control uses loudspeakers and error microphones in the passenger cabin or flight deck in conjunction with a control system. Active structural acoustic control uses actuators coupled to the structure along with error microphones. Passive noise control, by contrast, seeks to reduce cabin noise by the modification of the airframe structure itself (e.g., sidewall construction detail).

Designing quiet structures is itself a form of passive noise control, and falls under the broad category of multidisciplinary design optimization (MDO).[1] Design of quiet aircraft cabins is multidisciplinary in that it requires satisfaction of not only acoustic goals and constraints, but also structural goals and constraints such as weight, stress, and aeroelastic effects. This makes the multidisciplinary design process extremely complicated, requiring many iterations of time-consuming analyses coupled with a mathematical programming algorithm as a means of optimizing the system. In addition, because of the complexity and often high nonlinearity of MDO problems, it is nearly impossible to guarantee that a solution is truly optimal. It remains the task of the designer to choose from among a set of candidate solutions the one that best satisfies not only the goals and constraints defined in the MDO problem, but also meets more intangible standards such as design experience and intuition. Although MDO may be difficult to implement, it offers a number of far-reaching benefits to the designer: 1) goal-orientation of the design approach; 2) tedium reduction; and 3) decision-making help, trend finding, and preliminary design guidance.[2]

Some examples of the use of automated structural optimization tools related to the previous text have appeared in the literature.[3–5] There is only limited work in the archival literature for explicit structural acoustic optimization. One example of such work is that of Lamancusa.[6] The literature review contained in the work of Hambric[7] provides a comprehensive and extensive look at the current art. Briefly, much of the structural

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acoustic optimization has been performed without explicit consideration of the acoustics, but, rather, by such means as avoiding structural resonances within a frequency band of interest. Further, in contrast to the work at hand, much of such work has used special-purpose analysis tools. Interested readers are referred to the work of Hambric[7] for more extensive discussion of such methods.

In the work at hand, objective functions similar to those used by Lamancusa[6] are implemented within an optimization algorithm applied to the design of a cylinder for minimization of interior noise. The algorithm is implemented by a computational design tool employing generally available modeling codes.[8] The tool integrates structural and acoustic analysis packages with an optimization algorithm as part of an automated process. The heart of the design tool is a UNIX shell script that automates the iterative analysis and optimization procedure. The user provides a group of input data files describing the structural and acoustic models and the parameters of the optimization procedure, and the design tool returns updated data files describing the optimized sound field and design of the structural model. The purpose of the overall research project is to develop and validate a flexible, robust tool for the optimal design of stiffened or unstiffened cylinders subject to various acoustic and structural objectives and constraints and excited by an external harmonic noise source.

The following sections introduce the computational structure of the design tool, and describe the specific models to which the optimization formulations are applied. Next, the four optimization formulations considered in this paper are introduced. The subsequent sections present the results pertaining to the formulations' performance, followed by our conclusions.

Description of the Design Tool

As stated earlier, the computational structural acoustic design tool is intended to be robust and flexible. The heart of the design tool is a modular UNIX Korn shell script. The shell controls the interaction of three main programs and three supporting codes and also provides error-trapping capabilities. Figure 1 is a flowchart depicting the interaction of the analysis and supporting programs within the shell.

The optimization uses the modified method of feasible directions algorithm to optimize a single objective function subject to inequality constraints and side constraints on the design variables (CONMIN).[9,10] The user provides an input file containing parameters that define convergence criteria, constraint tolerances, and upper bounds on array dimensions needed for the optimizer. The convergence criteria used here are as follows: for five consecutive iterations the relative change in the normalized objective function is less than 1%, or the absolute change is less than 0.001.

A finite element method (FEM) code is used for the analyses of the structural model (MSC/NASTRAN Version 68). All structural models are assumed to have structural damping. The user is required to prepare a FEM input dataset describing the structural model and the external loading conditions. These external loads are acoustic pressure loads computed using an acoustic boundary element method (BEM) code. Solution of the external sound field is required prior to launch of the design tool, as the FEM input dataset requires the corresponding load data.

The same acoustic BEM code (COMET/Acoustics) used to obtain the exterior pressure loading is also used within the design tool to perform the acoustic analyses of the cylinder interior. The particular acoustic BEM implementation uses the indirect boundary element method for these analyses. The acoustic analysis for the cylinder models is uncoupled, meaning that the vibration of the structure is not considered to be affected by the bounding acoustic medium. The user is required to provide an acoustic BEM input dataset, which defines not only the boundary elements on the inner surface of

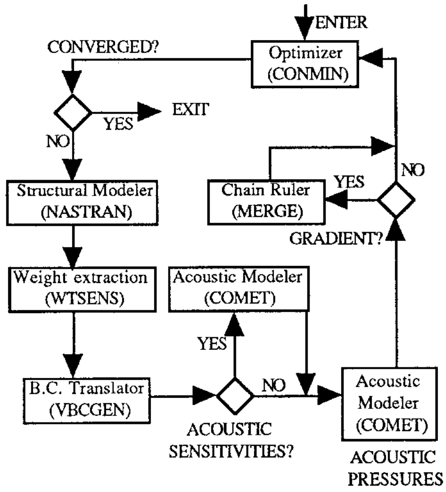

Fig. 1 Schematic of design tool algorithm logic.

the cylinder, but also the position of interior data recovery nodes.

The VBCGEN program translates structural surface velocities from the structural FEM modeler into a format readable by the acoustic BEM for use as velocity boundary conditions. The acoustic BEM code produces sensitivities of the recovered pressures to changes in the velocity boundary conditions. These acoustic sensitivities are combined with the FEM modeler's structural sensitivities to produce structural acoustic system sensitivities. The acoustic sensitivities need to be computed only once for each cylinder model. The program WTSENS is used to determine the total cylinder weight and weight sensitivities from the FEM output files. These are the sensitivities of the structural surface velocities to changes in the design variables. The MERGE program combines the structural sensitivities and the acoustic sensitivities into global system sensitivities

$$\frac{\partial \hat{p}_i}{\partial x_j} = \frac{\partial \hat{p}_i}{\partial v}\frac{\partial v}{\partial x_j} \tag{1}$$

where v is the surface normal velocity on the interior of the structure, $\partial \hat{p}_i/\partial v$ is the sensitivity produced by the acoustic BEM, and $\partial v/\partial x_j$ is the sensitivity produced by the structural FEM. The optimizer requires gradient information initially and again every three or four function evaluations, and so MERGE does not run on every loop through the design tool.

Cylinder Models

FEMs for an unstiffened isotropic cylinder are used for the research presented here. The physical dimensions of the cylinder models are the same as the outer shell of the model used by Grosveld et al.[11] in their study of active structural acoustic noise control. The models are fully defined by their input data files. The cylinder is considered to be made of aluminum, 3.66 m in length with a radius of 0.838 m. Upper and lower bounds on shell thickness are 1.19 and 2.21 mm, respectively, and the shell has an initial design thickness of 1.7 mm. Both ends of the cylinder are assumed to be clamped. The cylinder is excited by a single exterior monopole source at a frequency of 157.9 Hz. This excites a (2,1) structural resonance of the cylinder. Note that this on-resonance condition exists only for the cylinder at its initial design thickness. The source is located 0.168 m to one side of the cylinder, halfway down its length. The acoustic loads produced by this source on the cylinder are computed by the acoustic BEM code and included in the FEM dataset. Since only symmetric modal response is being investigated at this time, analysis is simplified by modeling only one quarter of the cylinder. The interior acoustic field is mod-

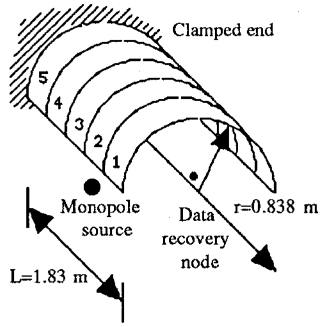

Fig. 2 Five-variable unstiffened isotropic cylinder model (not to scale).

eled by a single acoustic data recovery point, located 1.7 m from the clamped end and just off of the cylinder axis.

Three finite element models for the cylinder are considered in this work: a 40-element, five-design variable coarse model, and two 800-element, 20-design variable fine models. The coarse FEM model is composed of 40 linear quadrilateral elements, grouped into five circumferential bands of eight elements each. The design variables represent the thickness of the elements in each of these bands and are numbered consecutively from 1 (the free end) to 5 (the clamped end). Figure 2 provides a simple diagram of the coarse model.

It is anticipated that trends in the performance of the formulations when applied to the coarse model will be reflected in the optimization of more detailed models. The dimensions of the fine models are the same as for the coarse model, except for the bounds on the thickness, which are set at 1.0 and 3.4 mm, with an initial design thickness of 1.7 mm. The fine models have 800 linear quadrilateral shell elements. One model has its 800 shell elements grouped into 20 circumferential bands. The thickness of each of these bands is a design variable. The other fine model has its structural elements grouped into 20 strips parallel with the axis. The fine models are assumed to be excited by a single monopole source, but at a frequency of 154.2 Hz. This frequency also corresponds to a (2,1) structural resonance. Again, note that the on-resonance condition only exists for the cylinder at the initial design thickness. The difference in the number of degrees of freedom between the coarse and fine models accounts for the difference in frequencies for the same structural resonance. The fine models have up to 188 acoustic data recovery nodes in the interior, rather than the single point of the coarse model.

Material properties employed for the analysis include air density of 1.21 kg/m^3, sound speed in air of 340 m/s, aluminum density of 2.7 kg/m^3, and Young's modulus of 71 GPa.

Optimization Problem Formulations

The objective function formulations used here are generic, in that they incorporate no explicit assumptions regarding the structure or acoustic environment. The simplest, least complex formulation, referred to hereafter as the baseline formulation, considers only the acoustics of a specified structure, whereas the other formulations consider both the acoustics and the weight of the structure. These additional formulations are referred to as the acoustic, weight, and compound formulations, where the names refer to the property minimized. In this sense, compound implies that acoustics and weight are being minimized simultaneously. All analyses are performed at a single excitation frequency. The following four sections detail each of these formulations.

Baseline Formulation

The objective of the baseline formulation is to minimize the sum of the squares of the acoustic pressure amplitudes at discrete data recovery points. The only constraints are side constraints bounding the design variables. Stated mathematically, the optimization problem is

Minimize

$$f(x) = \sum_{i=1}^{\mathrm{NDRN}} \hat{p}_i(x)\hat{p}_i^*(x) \Big/ f_0 \tag{2}$$

Subject to

$$x_L \leq x \leq x_U \tag{3}$$

In the objective function, Eq. (2), $\hat{p}_i(x)$ is the complex pressure at the ith data recovery node, the asterisk indicates complex conjugation, and NDRN is the number of data recovery nodes within the volume of interest. The objective function is scaled by f_0, the value of the objective at the initial design state. Lower and upper bounds on the design variables are represented by x_L and x_U, respectively. Note that for the coarse model, described earlier, the summation in Eq. (2) is for a single data recovery node.

Acoustic Formulation

This formulation seeks the vector of design variables that minimizes the sum of the squared acoustic pressures subject to the side constraints and to a constraint on the total weight of the structure. In a similar manner, Lamancusa[6] investigated optimizing the thickness of an isotropic plate to minimize radiated acoustic power, subject to a constraint on the mass of the plate. The formulation used here is

Minimize

$$f(x) = \sum_{i=1}^{\mathrm{NDRN}} \hat{p}_i(x)\hat{p}_i^*(x) \Big/ f_0 \tag{4}$$

Subject to

$$(W/W_{\mathrm{max}}) - 1 \leq 0 \tag{5}$$

$$x_L \leq x \leq x_U \tag{6}$$

In Eq. (4), f_0 is the initial value of the objective function. In Eq. (5), W is the total weight of the structure, and W_{max} is a user-specified maximum weight. It is entirely possible that the initial weight of the structure will exceed the specified maximum, indicating an infeasible starting point. This is not a problem for the tool's optimizer, which is usually able to rapidly move the design into the feasible space.

Weight Formulation

This formulation seeks the vector of design variables that minimizes the weight of the structure subject to the side constraints and a constraint on the maximum value of the sum of the squared pressures. Again, Lamancusa[6] investigated a similar formulation for a plate model, where mass was minimized subject to a constraint on the radiated acoustic power. The weight formulation is stated as

Minimize

$$f(x) = W/W_0 \tag{7}$$

Subject to

$$\left[\sum_{i=1}^{\mathrm{NDRN}} \hat{p}_i(x)\hat{p}_i^*(x) \Big/ \left(\sum |\hat{p}|^2\right)_{\mathrm{max}}\right] - 1 \leq 0 \tag{8}$$

$$x_L \leq x \leq x_U \tag{9}$$

In Eq. (7), W_0 is the initial value of the weight, whereas in Eq. (8), $(\Sigma\ |\hat{p}|^2)_{max}$ is the maximum sum of the squared pressures. In the actual implementation of Eq. (8), the user specifies a desired decibel level for this maximum, and the algorithm converts this level to an appropriate sum of squared pressure magnitudes. The weight formulation is in effect the converse of the acoustic formulation, and it will be observed that they produce very similar results.

Compound Formulation

This formulation is an attempt to find the vector of design variables that simultaneously minimizes the sum of the squared acoustic pressures and the structural weight. Because the tool's optimizer can only minimize a single objective function, it is necessary to find a way to cast this multiple objective problem into a single objective problem.[12] The approach taken here is to introduce auxiliary design variables β_a and β_w that will act as upper bounds on the acoustic pressure and structural weight, respectively. The actual weight and pressure are then incorporated in the formulation as constraints bounded by β_a and β_w. The actual objective function is then the weighted sum of β_a and β_w, where the weights, μ_a and μ_w, permit the user to place greater significance upon one element of the sum (pressure or weight). The mathematical statement of the formulation is

Minimize

$$\mu_a\beta_a + \mu_w\beta_w \tag{10}$$

Subject to

$$\left[\sum_{i=1}^{NDRN} \hat{p}_i(x)\hat{p}_i^*(x) \bigg/ \left(\sum |\hat{p}|^2\right)_{ref}\right] - \beta_a \leq 0 \tag{11}$$

$$(W/W_{ref}) - \beta_w \leq 0 \tag{12}$$

$$x_L \leq x \leq x_U \tag{13}$$

where $(\Sigma\ |\hat{p}|^2)_{ref}$ is the initial value of the sum of the squared pressures, and similarly, W_{ref} is the initial weight of the structure. Notice that this method allows additional objectives to be modeled, merely by adding additional beta variables and constraints.

Results

The performance of each of the formulations is evaluated by optimizing the coarse cylinder model under a variety of constraint conditions and from three different initial starting points in the design space. Multiple initial designs are needed since there is no guarantee that a single global optimal solution exists for each set of objectives and constraints. Indeed, the existence of multiple local minima, each representing a local optimum, is a distinct probability. As mentioned in the discussion of the cylinder models, the standard starting design for the coarse model is a uniform thickness of 1.7 mm for each of the design variables, corresponding to a weight for the quarter cylinder of 218 N (hereafter, weight will imply weight of the quarter cylinder.) The other two starting designs are chosen to set all of the design variables either to their upper or lower bounds. Table 1 summarizes these three initial design states. In the remainder of this paper, the starting design states will be referred to as the upper, middle, and lower cases, corresponding to all design variables at the upper bound, intermediate state, and lower bound, respectively.

Care must be taken when comparing functions based on the pressures at data recovery nodes, for both coarse and fine models. The excitation amplitude used here is arbitrary, and the models themselves have vastly different levels of refinement. Therefore, the initial interior levels between the two models, for the same exterior forcing, cannot be expected to be equal. Further, the initial level and reduction from this initial level is of limited value, as the initial value is a strong function of the initial design state. Yet the choice of an excitation frequency that produces an on-resonance condition at the standard design state is of no particular importance to the method or the results. Indeed, as soon as a single design variable differs from the standard thickness, the on-resonance condition no longer pertains. It is important to understand, then, that the ending levels are what is significant and comparable between formulations. Therefore, an arbitrary 0 dB reference was selected and all starting and ending points are expressed with respect to this reference.

Table 1 Initial design states, coarse model

Initial state	Design variables, mm	Cylinder weight, N	SPL, dB
Upper	2.21	283.2	−5.2
Middle	1.70	217.9	−2.9
Lower	1.19	152.5	0.2

Table 2 Results of coarse model optimization with baseline formulation

Design state	Objective function, dB	
	Initial	Final
Upper	−5.2	−14.8
Middle	−2.9	−14.9
Lower	0.2	−14.8

Baseline Formulation

The baseline formulation was run for each of the initial design states. Table 2 presents the results of these optimization runs. The observation that the final values of the objective function are essentially the same for each case suggested that they have each converged to the same final design. This is in fact the case. The weight of the optimal cylinder for these cases is 231 N. The first, second, and third design variables are at the upper bound of 2.21 mm, whereas the fourth and fifth design variables are at the lower bounds of 1.19 mm. The optimal design has increased thickness toward the free end of the quarter cylinder (the center of the full cylinder) and reduced thickness at the clamped end. The weight of the cylinder at the optimal design point, because it is not constrained, has increased by 5.9% over the weight of the standard cylinder.

Acoustic Formulation

Each of the three remaining formulations considers the weight of the cylinder, requiring more cases to explore the behavior of these formulations. Eight constraint conditions on the maximum permissible cylinder weight are summarized in Table 3, where constraint case 3 indicates a model where the cylinder weight is not allowed to increase over the weight of the standard cylinder. Eight constraint cases and three design starting points yield 24 different combinations of initial design and maximum weight constraint. For each constraint case, the model is started from the same three initial design points as for the baseline formulation. Though the upper starting design point, with a weight of 283 N, is initially infeasible for each constraint case, as is the middle starting design point for cases 4–8, the optimization algorithm is capable of bringing these initially infeasible design states into the feasible region. Figure 3 is a plot of the results of 24 tests of the acoustic formulation of the coarse cylinder optimization.

Table 3 Constraint conditions, acoustic formulation

Constraint case	Constrained maximum weight, N	Standard cylinder weight, %
1	240	110
2	229	105
3	218	100
4	207	95
5	196	90
6	185	85
7	174	80
8	163	75

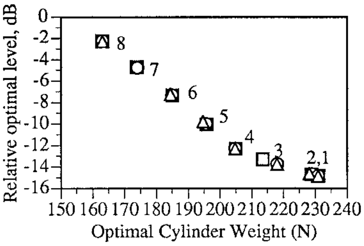

Fig. 3 Relative optimal level vs optimal weight, coarse model, acoustic formulation: ○, upper initial state; □, middle initial state; and △, lower initial state.

There appears to be a nearly linear relationship between the optimal weight and the optimal decibel sound pressure level at the data recovery node in the coarse cylinder. No comparable result has been found in the literature that relates the sound pressure level and weight in this manner for cylinders. However, the mass law for predicting transmission loss through panels at low frequencies would provide such a linear slope. This aspect will be addressed in greater detail later.

The eight groupings of optimal design points closely correspond to the eight constraint cases. The number by each group of data points in Fig. 3 indicates the constraint case that yielded that grouping. The optimal designs for cases 8 through 4 are at their constrained maximum weights and have converged to the same ending design. For case 3, which is constrained to the weight of the standard cylinder, the middle initial design point yielded a slightly different optimal design than the upper and lower points. The middle state yielded an optimal weight of 213.5 N, whereas the upper and lower states yielded designs that weigh 218 N. The six designs for cases 2 and 1 have converged to almost the same point around 230 N and −15 dB, but case 2 is constrained to weigh no more than 229 N. The 230 N, −15 dB design point corresponds exactly to the design found with the baseline formulation, and the vector of design variables have the same values. The 230 N weight is well below the constraint of 240 N for case 1, although it was still within the optimizer's constraint tolerance of −0.1, so that the constraint is considered active.

As with the baseline formulation, the optimal state indicates that the cylinder thickness is at the lower bound at the clamped end and increases toward the free end. Table 4 lists the optimal design states for four of the design constraint cases. Within each constraint case, except for case 3, the three starting points converged to essentially the same ending state for that case (indirectly indicated by the clustering of the points in Fig. 3). The upper and lower starting design states for case 3 yielded the same ending state, which is different from the ending state for the middle starting state.

Weight Formulation

The weight objective was evaluated using six different constraint cases. The weight is minimized with a constraint on the maximum sum of the squared pressures. A maximum value is specified for the average sound pressure level at the data recovery nodes. This is multiplied by the number of nodes to determine the maximum sum of the squared pressures. Table 5 lists the six constraint conditions. Case 2, with a maximum sound pressure level (SPL) of −3 dB, essentially represents the standard (1.7-mm-thick) cylinder. The upper initial design point is infeasible with respect to the constraints of cases 3–6. The middle initial design point is infeasible for cases 3–6, and for case 2 the constraint is active. The lower initial design point is infeasible for constraint cases 2–6 and active for case 1.

Table 4 Final design variable vector, coarse model, acoustic formulation, constraint cases 1, 3, 5, 7, middle initial design

Constraint	Design variables, mm				
case	1	2	3	4	5
1	2.21	2.21	2.21	1.19	1.19
3	2.21	2.21	1.53	1.19	1.19
5	1.86	2.21	1.19	1.19	1.19
7	1.65	1.56	1.19	1.19	1.19

Table 5 Constraint conditions, weight formulation

Constraint case	Constrained maximum SPL, dB
1	0
2	−3
3	−6
4	−9
5	−12
6	−15

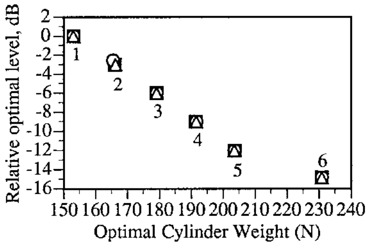

Fig. 4 Relative optimal level vs optimal weight, coarse model, weight formulation: ○, upper initial state; □, middle initial state; and △, lower initial state.

Figure 4 presents the results of the optimization of the coarse model using this objective and constraint formulation. These data again show the nearly linear relationship between optimal decibel level at the node and optimal weight. As with the acoustic formulation, the optimal solutions for the six constraint cases are distinct from each other. For the weight formulation, the three initial design states for all cases except case 2 converge to the same optimal state. As with constraint case 3 of the acoustic formulation, the middle design for case 2 here converges to a slightly different state than the upper and lower initial designs. The end effect of this difference is barely discernible in the cluster of points at 160 N in Fig. 4. For all the models, the pressure constraint is active for every optimal design, and well within the optimizer's constraint tolerance. Again, the optimal cylinder skin thickness distribution placed increased thickness at the free end of the cylinder, with minimum thickness at the clamped end.

Compound Formulation

For the compound formulation, the objective rather than the constraints was varied to establish seven optimization cases.

Table 6 Weighting coefficients, compound formulation

Case	μ_a	μ_w
1	1	1
2	2	1
3	3	1
4	4	1
5	10	1
6	100	1
7	1000	1

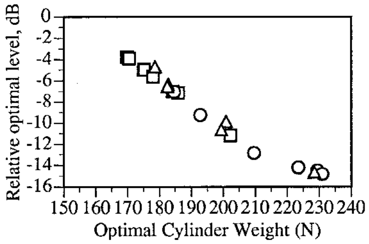

Fig. 5 Relative optimal level vs optimal weight, coarse model, compound formulation: ○, upper initial state; □, middle initial state; and △, lower initial state.

Recall from Eq. (10) that the user is free to specify weighting coefficients to the acoustic and weight terms of the objective function. Table 6 lists the acoustic and weight weighting coefficients, μ_a and μ_w, respectively, chosen for each of the seven cases. Since the initial values of the beta design variables are set to one, the constraints for all of the models are initially active. Figure 5 presents the results of the compound formulation optimizations. Once again a linear relationship is observed between the decibel level and the weight. However, the results of the three initial design states for each objective case do not remain clustered as they do for the acoustic and weight formulations. An example is objective case 1, where the upper initial design converged to −7.1 dB at 185 N, the middle initial design converged to −3.8 dB at 170 N, and the lower initial design converged to −9.9 dB at 201 N. Recall that the objective function for this case is to minimize the weighted sum of the acoustic and weight beta variables. The optimization terminates when the change in the sum of the beta variables is less than the convergence criterion, not necessarily when the sum of the pressures or the weight is at a minimum. On the iterations where the acoustic constraint is active, the weight constraint is inactive. This indicates that the two objectives, minimum pressure and minimum weight, are conflicting and that it is not possible to simultaneously minimize both. Note that this conflict can be inferred from Figs. 3–5, where a low SPL requires a high weight.

Optimization of the 20-Variable Model

Results for the 20-variable cylinder models will be briefly presented here. Fewer cases are considered with the fine models, as compared to the number of cases for the coarse models. Total execution time for the 20-variable model can be up to 40 times that for the five-variable model. This is a consequence of the greater computational effort required to perform the structural and acoustic analyses and the greater number of data recovery nodes in the fine models.

The acoustic, weight, and compound formulations were used to optimize the models. The results are plotted in Fig. 6 along with all the previous results of the coarse model. Figure 6 also indicates the relative interior levels for coarse and fine models at different weights with uniform thickness distribution, and the noise reduction predicted using the mass law. The mass law may be posed here as

$$\Delta_{\mathrm{dB}} = -20 \log(m/m_{\mathrm{ref}}) \tag{14}$$

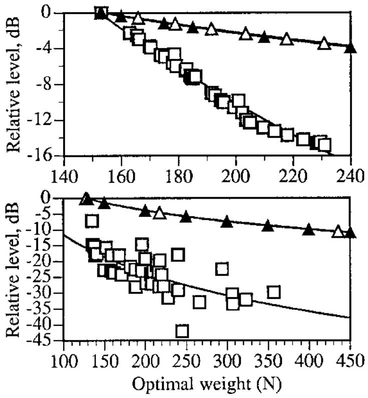

Fig. 6 Relative optimal level vs optimal weight, coarse and fine models, acoustic, weight, and compound formulations. Upper plot, coarse model; lower plot, fine model: □, optimized; △, unoptimized, uniform thickness; and , mass law.

where Δ_{dB} is the relative change in noise level at fixed frequency for a cylinder of mass m compared to a cylinder of mass m_{ref}. The mass law predicts a 6-dB reduction in level for each doubling of mass. The lines in Fig. 6 represent logarithmic curve fits to the uniform thickness and optimized results. Clearly, the uniform thickness cylinder results exhibit reductions consistent with the mass law, as might be expected in this low-frequency regime. The curve fit to the uniform thickness data for the coarse model yields a 5.9-dB reduction per doubling of mass, very close to mass law behavior. The fit to the uniform thickness data for the fine model effectively yields 6-dB reduction per doubling of mass, or mass law behavior. The curve fit to the optimized thickness data for the coarse model yields 25.9-dB reduction per doubling of mass, significantly greater than mass law. The curve fit to the optimized thickness data for the fine model yields 12.3-dB reduction per doubling of mass, again greater than mass law. Note that the goodness of fit for the fine model results is lower than for the coarse model results.

The relative levels for the uniform cylinders are higher than for cylinders of the same weight with an optimal thickness distribution. This clearly demonstrates the value of optimization in passive noise control for cylinders. A 20-variable cylinder with an optimal thickness distribution weighing 217 N (the weight of the standard cylinder) has an average level that is 20 dB lower than a uniform cylinder of the same weight. It is evident, then, that adding skin thickness uniformly will have less impact on the internal acoustic environment, as compared to an optimal thickness distribution. Indeed, the difference between the slopes of the fit to the uniform thickness data and that of the optimized data yields a measure of the difference between the normal use of mass and the effective, optimal use of mass, with the design optimization process finding this optimal use of mass.

In general, there is greater variation in the results produced by the fine model than for the coarse model. The fine model, with 20 times as many structural shell elements, captures more of the structural behavior of the cylinder, and the greater number of acoustic data recovery nodes provides the optimizer with more detailed gradient information. The fine models take longer to run and require more iterations to converge to an optimum, but the acoustic and weight formulations are still faster than the compound formulation. The numerical instability encountered previously for the weight formulation also occurred for some of the weight and compound formulation cases of the fine model, but not for the acoustic formulation cases. This lends further credence to our position that the acoustic formulation is the most effective.

Initial execution times for the fine model required between 12–20 iterations through the optimizer, and consumed on the order of 36 h of wall clock time on a multiuser DEC ALPHA. A great deal of effort has been expended for improving the computational efficiency of the overall algorithm. Current optimizations require the same number of iterations, but consume less than 12 h of clock time.

Comparison of Formulations

The optimization problem formulations produce very similar results in terms of optimal cylinder design. As noted earlier, the optimal decibel levels produced by each formulation appear to be linear with roughly the same slope. Logarithmic fits, as described in the previous section, to the data produced by each formulation, yield essentially the same leading constant. This result, as emphasized earlier, must be interpreted with care. Merely adding weight to the cylinder does not guarantee reduced sound pressure levels in the interior. The shell thickness distribution of the cylinder is the most important factor. Recall the optimal design of acoustic formulation case 1 for the coarse cylinder, which weighs about 230 N, even though the weight constraint allows up to 240 N. Because the optimal design variables are at the upper bounds where thickness is increased, any additional weight will have to be added to the thin end of the cylinder and will probably increase the SPL. Therefore, the determining factors in how much the sound pressure can be reduced in the cylinder are the bounds on the design variables first, and then the total constrained weight.

One question that still remains is which, if any, of the formulations is faster and more reliable in finding an optimal solution? The time required for the analysis codes to run is the same at each iteration for each formulation, and so the critical indicator of speed is the number of iterations required to converge to an optimal solution. Table 7 presents iteration statistics for each of the formulations. The statistics indicate that the baseline formulation tends to be the fastest, although it must be remembered that the baseline formulation does not consider the cylinder weight. Of the three other formulations, the weight formulation, on average, converges to an optimal solution faster than the other two. The acoustic formulation is nearly as fast. The compound formulation has significant scatter in the number of iterations required to converge, and it requires more iterations than the acoustic or weight formulations.

The weight formulation is the fastest of the four formulations. However, the optimizer suffers from numerical instability on some of the cases where the weight formulation is used. For 8 of the 18 tests, the optimizer violated the side constraints on the design variables. The FEM modeler then aborted when presented with a cylinder design that violated physical bounds (negative thickness). The source of this problem has not been located, although it has been determined that it is internal to the optimizer code, and not caused by, in some way, the weight formulation subroutines. By applying additional scaling to the gradients of the objective and constraints, the numerical instability is avoided. This method gives results consistent with the cases where no instability is encountered. In light of this, it seems that the acoustic formulation is preferred, being both fast and reliable. The weight formulation tends to be unreliable, and the compound formulation is slow and inconsistent.

Table 7 Iteration statistics, coarse model, baseline, acoustic, weight, and compound formulations

Formulation	Number of models	Number of iterations: Average	Standard deviation
Baseline	3	7.33	1.53
Acoustic	24	9.26	2.47
Weight	18	7.78	1.63
Compound	21	13.00	7.11

Conclusions

The principal objective of the work considered here has been the implementation of a structural acoustic design optimization algorithm, and the evaluation of specific objective functions within that algorithm. Of the objective functions evaluated here, we conclude that the acoustic formulation is the best formulation. It is the most reliable and is also relatively fast. It produces consistent results, with most of the designs using the maximum weight of material available. Its approach to the design problem seems the most reasonable. It attempts to optimize the acoustics subject to a weight constraint. In a more complicated problem, additional structural constraints will exist that will have to be satisfied.

Within the greater context of the algorithm development and evaluation, the results obtained from the example problems are suggestive that significant noise reduction potential exists for tonal excitations. The results obtained from the tests of the optimization formulations that there is a nearly logarithmic relationship, akin to the mass law but with greater impact, between the decibel sound pressure level for this unstiffened cylinder model and the total weight of the cylinder at an optimal design state, where skin thickness is the design variable. This relationship is true especially for the coarse model, although the general trend is also exhibited by the fine model. It is important to keep in mind, however, that it is only through optimal distribution of the skin thickness that the greatest noise level reduction can be achieved. A cylinder with uniform distribution of thickness will not have as significant a noise reduction. We must also re-emphasize that the particular structures evaluated here are not representative of actual fuselage construction.

There is great similarity in the optimal vectors of design variables for the coarse model. The tendency is for the design variables at the clamped end to be at their minimum bounds, with thickness increasing toward the free end. For the fine circumferential model, some of the models have this type of optimal cylinder thickness distribution, although some models also place greater thickness near the clamped end. While no one solution to the cylinder design problem can be declared the true optimum, the trends observed can serve as heuristics for the optimal structural acoustic design of cylinders. There are a few obvious limitations to this. First, the unstiffened models are not representative models for an aircraft fuselage. Second, a cylinder with a stepwise or even a gradual change in thickness along its length or circumference would be difficult to manufacture. Optimizing the size and location of stiffeners may be more realizable from a manufacturing standpoint. However, the results of this research indicate that noise reductions can be achieved with relatively few stepwise changes in skin thickness, and this may become a practical design technique in future aircraft. Finally, and perhaps most importantly, the design optimization of an aircraft fuselage will be driven by structural considerations relating to aeroelasticity and performance. This research demonstrates that such optimization is possible. By adding constraints to model other considerations such as stress limitations, an accurate fuselage model can be optimized.

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